

Nilpotent Symmetries For A Free Relativistic Particle In Augmented Superfield Formalism

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Abstract: In the framework of the augmented superfield formalism, the local, covariant, continuous and off-shell (as well as on-shell) nilpotent (anti-)BRST symmetry transformations are derived for a $(0 + 1)$ -dimensional free scalar relativistic particle that provides a prototype physical example for the more general reparametrization invariant string- and gravitational theories. The trajectory (i.e. the world-line) of the free particle, parametrized by a monotonically increasing evolution parameter τ , is embedded in a D -dimensional flat Minkowski target manifold. This one-dimensional system is considered on a $(1 + 2)$ -dimensional supermanifold parametrized by an even element τ and a couple of odd elements (θ and $\bar{\theta}$) of a Grassmannian algebra. The horizontality condition and the invariance of the conserved (super)charges on the (super)manifolds play very crucial roles in the above derivations of the nilpotent symmetries. The geometrical interpretations for the nilpotent (anti-)BRST charges are provided in the framework of augmented superfield approach.

Keywords: Augmented superfield formalism; (anti-)BRST symmetries; free scalar relativistic particle; horizontality condition; invariance of (super)charges on (super)manifolds

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1 Introduction

The principle of local gauge invariance, present at the heart of all *interacting* 1-form gauge theories, provides a precise theoretical description of the three (out of four) fundamental interactions of nature. The *crucial* interaction term for these interacting theories arises due to the coupling of the 1-form gauge fields to the conserved matter currents so that the local gauge invariance could be maintained. In other words, the requirement of the local gauge invariance (which is more general than its global counterpart) enforces a theory to possess an interaction term (see, e.g., [1]). One of the most attractive approaches to covariantly quantize such kind of gauge theories is the Becchi-Rouet-Stora-Tyutin (BRST) formalism where the unitarity and “quantum” gauge (i.e. BRST) invariance are respected together at any arbitrary order of the perturbation theory (see, e.g., [2]). The BRST formalism is indispensable in the context of modern developments in topological field theories, topological string theories, supersymmetric gauge theories, reparametrization invariant theories (that include D-branes and M-theories), etc., (see, e.g., [3-5] and references therein for details).

We shall be concentrating, in our present investigation, only on the geometrical aspects of the BRST formalism in the framework of augmented superfield formulation because such a study is expected to shed some light on the abstract mathematical structures behind the BRST formalism in a more intuitive and illuminating fashion. The usual superfield approach [6-13] to the BRST scheme provides the geometrical origin and interpretation for the conserved ($\dot{Q}_{(a)b} = 0$), nilpotent ($Q_{(a)b}^2 = 0$) and anticommuting ($Q_b Q_{ab} + Q_{ab} Q_b = 0$) (anti-)BRST charges $Q_{(a)b}$ which generate local, covariant, continuous, nilpotent ($s_{(a)b}^2 = 0$) and anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$) (anti-)BRST transformations $s_{(a)b}$ for *only* the gauge field and (anti-)ghost fields of an interacting gauge theory. This is achieved by exploiting the so-called horizontality condition [6-13] (which has been christened as the “soul-flatness” condition in [14]). In fact, these attempts (see, e.g., [6-14]) have been primarily made to gain an insight into the existence of a possible connection between the ideas behind the supersymmetries and the (anti-)BRST symmetries. As a bonus and by-product, one obtains the nilpotent (anti-)BRST symmetry transformations for the gauge- and the (anti-)ghost fields for the (anti-)BRST invariant Lagrangian density of a given gauge theory. The horizontality condition requires the $(p+1)$ -form super-curvature, defined on the $(D+2)$ -dimensional supermanifold, *to be equal* to the $(p+1)$ -form ordinary curvature for a p -form ($p = 1, 2, 3, \dots$) gauge theory, defined on an ordinary D -dimensional spacetime manifold. In the above, the $(D+2)$ -dimensional supermanifold is parametrized by the D -number of spacetime *even* coordinates and two Grassmannian *odd* coordinates. Basically, the horizontality condition owes its origin to the (super)exterior derivatives $(\tilde{d})d$ (with $\tilde{d}^2 = 0, d^2 = 0$) which are one of the three (super) de Rham cohomological operators. In a set of papers [15-19], all the three (super)cohomological operators have been exploited to derive the (anti-)BRST symmetries, (anti-)co-BRST symmetries and a bosonic symmetry for the two-dimensional free Abelian gauge theory in the superfield formulation where the generalized versions of the

horizontality condition have been exploited. All the above attempts [6-19], however, have *not* yet been able to shed any light on the nilpotent symmetries that exist for the *matter* fields of an *interacting* gauge theory. Thus, the results of the above approaches [6-19] are still partial as far as the derivation of *all* the symmetry transformations are concerned.

Recently, in a set of papers [20-22], the restriction due to the horizontality condition has been augmented with the requirement of the invariance of matter (super)currents on the (super)manifolds [†]. The latter restriction produces the nilpotent (anti-)BRST symmetry transformations for the matter fields of a given interacting gauge theory. The salient features of these requirements are (i) there is a beautiful consistency and complementarity between the nilpotent transformations generated by the horizontality restriction and the requirement of conserved matter (super)currents on the (super)manifolds. (ii) The geometrical interpretations for the (anti-)BRST charges $Q_{(a)b}$ as the translation generators $(\text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta))\text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$ along the $(\theta)\bar{\theta}$ -directions of the $(D+2)$ -dimensional supermanifold remain intact. (iii) The nilpotency of the (anti-)BRST charges is encoded in a couple of successive translations (i.e. $(\partial/\partial\theta)^2 = (\partial/\partial\bar{\theta})^2 = 0$) along either of the two Grassmannian directions of the supermanifold in the framework of both the restrictions. (iv) The anticommutativity of the (anti-)BRST charges (and the transformations they generate) is captured in the relationship $(\partial/\partial\theta)(\partial/\partial\bar{\theta}) + (\partial/\partial\bar{\theta})(\partial/\partial\theta) = 0$ for the validity of both the restrictions. Thus, it is clear that both the above restrictions enable us to obtain all the nilpotent symmetry transformations for *all* the fields of an *interacting* gauge theory.

The purpose of the present paper is to derive the nilpotent (anti-)BRST transformations for *all* the fields present in the description of a free scalar relativistic particle (moving on a world-line) in the framework of augmented superfield formulation [20-22]. This study is essential primarily on three counts. First, to check the mutual consistency and complementarity between (i) the horizontality condition, and (ii) the invariance of the conserved (super)charges on the (super)manifolds for this physical system. These were found to be true in the case of interacting (non-)Abelian gauge theories in two $(1+1)$ -dimensions (2D) and four $(3+1)$ -dimensions (4D) of spacetime [20-22]. Of course, the latter theories were considered on the four $(2+2)$ -dimensional- and six $(4+2)$ -dimensional supermanifolds, respectively. Second, to generalize our earlier works [20-22] (which were connected *only* with the gauge symmetries) to the case where the gauge symmetries as well as the reparametrization symmetries co-exist in the theory. Finally, to tap the potential and power of the above restrictions for the case of a new physical system defined on a new three $(1+2)$ -dimensional supermanifold which is somewhat different from our earlier considerations of the interacting (non-)Abelian gauge theories on four $(2+2)$ - and six $(4+2)$ -dimensional supermanifolds.

The contents of our present paper are organized as follows. To set up the notations and conventions, we recapitulate the bare essentials of the (anti-)BRST symmetries for the free relativistic particle in Section 2. This is followed, in Section 3, by the derivations of the (anti-)BRST symmetries for the gauge (einbein)- and (anti-)ghost fields in the framework

[†]We christen this extended version of the usual formulation as the augmented superfield formalism.

of superfield formulation. Section 4 is devoted to the derivation of the above nilpotent symmetry transformations for the target space variables. Finally, we make some concluding remarks in Section 5 and point out a few future directions for our further investigations.

2 Nilpotent (anti-)BRST symmetries in Lagrangian formulation

Let us begin with the various equivalent forms of the gauge- and reparametrization invariant Lagrangians for a free scalar relativistic particle moving on a world-line that is embedded in a D -dimensional flat Minkowski target manifold. These specific Lagrangians are [23,24]

$$L_0 = m (\dot{x}^2)^{1/2}, \quad L_f = p_\mu \dot{x}^\mu - \frac{1}{2} e (p^2 - m^2), \quad L_s = \frac{1}{2} e^{-1} (\dot{x})^2 + \frac{1}{2} e m^2. \quad (2.1)$$

In the above, the mass-shell condition ($p^2 - m^2 = 0$) and the force free (i.e. $\dot{p}_\mu = 0$) motion of the *free* relativistic particle are a couple of common features for (i) the Lagrangian with the square root L_0 , (ii) the first-order Lagrangian density L_f , and (iii) the second-order Lagrangian L_s . Except for the mass (i.e. the analogue of the cosmological constant) parameter m , the target space canonically conjugate coordinates $x^\mu(\tau)$ (with $\mu = 0, 1, 2, \dots, D-1$) as well as the momenta $p_\mu(\tau)$ and the einbein field $e(\tau)$ are the functions of the monotonically increasing parameter τ that characterizes the trajectory (i.e. the world-line) of the free scalar relativistic particle. Here $\dot{x}^\mu = (dx^\mu/d\tau)$ are the generalized versions of the “velocity” of the particle. All the above dynamical variables as well as the mass parameter m are the *even* elements of the Grassmann algebra. The first- and the second-order Lagrangians are endowed with the first-class constraints $\Pi_e \approx 0$ and $p^2 - m^2 \approx 0$ in the language of Dirac’s classification scheme where Π_e is the canonical conjugate momentum corresponding to the einbein field $e(\tau)$. The existence of the first-class constraints on this physical system establishes the fact that this reparametrization invariant theory of the free relativistic particle is a *gauge* theory [‡]. For the covariant canonical quantization of such systems, one of the most elegant and suitable approaches is the Becchi-Rouet-Stora-Tyutin (BRST) formalism. The (anti-)BRST invariant Lagrangian L_B corresponding to the above first-order Lagrangian L_f is as follows (see, e.g., [24] and references therein)

$$L_B = p_\mu \dot{x}^\mu - \frac{1}{2} e (p^2 - m^2) + b \dot{e} + \frac{1}{2} b^2 - i \dot{\bar{c}} \dot{c}, \quad (2.2)$$

where the *even* element $b(\tau)$ is the Nakanishi-Lautrup auxiliary field and the (anti-)ghost fields $(\bar{c})c$ are the *odd* elements of the Grassmann algebra (i.e. $\bar{c}^2 = c^2 = 0, \bar{c}\bar{c} + c\bar{c} = 0$). The above Lagrangian density (2.2) respects the following off-shell nilpotent ($s_{(a)b}^2 = 0$)

[‡]It can be readily seen that under the infinitesimal transformations $\delta_r x_\mu = \epsilon \dot{x}_\mu, \delta_r p_\mu = \epsilon \dot{p}_\mu, \delta_r e = (d/d\tau)[(\epsilon e)]$ generated by the basic reparametrization $\tau \rightarrow \tau - \epsilon(\tau)$, the Lagrangian density L_f transforms to $\delta_r L_f = (d/d\tau)[(\epsilon L_f)]$. Similarly, under the gauge transformations $\delta_g x_\mu = \xi p_\mu, \delta_g p_\mu = 0, \delta_g e = \dot{\xi}$, the Lagrangian density L_f transforms to a total derivative. Both these transformations are equivalent (with the identification $\xi = e\epsilon$) for the free (i.e. $\dot{p}_\mu = 0$) relativistic particle because both the above transformations owe their origin to the mass-shell condition $p^2 - m^2 = 0$. It is evident that $\dot{p}_\mu = 0$ and $p^2 - m^2 = 0$ are a couple of salient features in the physical description of a free relativistic particle.

(anti-)BRST $s_{(a)b}$ symmetry transformations [§] (with $s_b s_{ab} + s_{ab} s_b = 0$) (see, e.g., [24])

$$\begin{aligned} s_b x_\mu &= c p_\mu, & s_b c &= 0, & s_b p_\mu &= 0, & s_b \bar{c} &= i\dot{b}, & s_b b &= 0, & s_b e &= \dot{c}, \\ s_{ab} x_\mu &= \bar{c} p_\mu, & s_{ab} \bar{c} &= 0, & s_{ab} p_\mu &= 0, & s_{ab} c &= -i\dot{b}, & s_{ab} b &= 0, & s_{ab} e &= \dot{\bar{c}}, \end{aligned} \quad (2.3)$$

because the Lagrangian density (2.2) transforms to the following total derivatives:

$$s_b L_B = \frac{d}{d\tau} \left[\frac{1}{2} c (p^2 + m^2) + b \dot{c} \right], \quad s_{ab} L_B = \frac{d}{d\tau} \left[\frac{1}{2} \bar{c} (p^2 + m^2) + b \dot{\bar{c}} \right]. \quad (2.4)$$

The on-shell ($\ddot{c} = 0, \ddot{\bar{c}} = 0$) nilpotent ($\tilde{s}_{(a)b}^2 = 0$) (anti-)BRST symmetry transformations $\tilde{s}_{(a)b}$ can be derived from (2.3) by the substitution $b = -\dot{e}$ as listed below

$$\begin{aligned} \tilde{s}_b x_\mu &= c p_\mu, & \tilde{s}_b c &= 0, & \tilde{s}_b p_\mu &= 0, & \tilde{s}_b \bar{c} &= -i\dot{e}, & \tilde{s}_b e &= \dot{c}, \\ \tilde{s}_{ab} x_\mu &= \bar{c} p_\mu, & \tilde{s}_{ab} \bar{c} &= 0, & \tilde{s}_{ab} p_\mu &= 0, & \tilde{s}_{ab} c &= +i\dot{e}, & \tilde{s}_{ab} e &= \dot{\bar{c}}, \end{aligned} \quad (2.5)$$

which turn out to be the symmetry transformations for the following Lagrangian density

$$\tilde{L}_B = p_\mu \dot{x}^\mu - \frac{1}{2} e (p^2 - m^2) - \frac{1}{2} (\dot{e})^2 - i \dot{\bar{c}} \dot{c}. \quad (2.6)$$

It should be noted that, for the on-shell nilpotent version of the (anti-)BRST transformations (2.5) and corresponding Lagrangian density (2.6), it is only the $b (= -\dot{e})$ field that has been replaced. Rest of the transformations of (2.3) remain intact. In the BRST quantization procedure, the first-class constraints $\Pi_e = b \approx 0$ as well as $p^2 - m^2 = -2\dot{b} \approx 0$ appear as the constraints on the physical states when one requires that the conserved and off-shell nilpotent BRST charge $Q_b = \frac{1}{2} c (p^2 - m^2) + b \dot{c} \equiv b\dot{c} - \dot{b}c$ should annihilate the physically meaningful states in the quantum Hilbert space of states. The conservation of the off-shell nilpotent ($Q_b^2 = 0$) BRST charge Q_b on any arbitrary *unconstrained* manifold is ensured by exploiting the equations of motion $\dot{p}_\mu = 0, \dot{b} = -\frac{1}{2}(p^2 - m^2), \ddot{c} = \ddot{\bar{c}} = 0, b + \dot{e} = 0, \dot{x}_\mu = e p_\mu$. The expression for the on-shell nilpotent BRST charge \tilde{Q}_b can be obtained from Q_b by the substitution $b = -\dot{e}$. Both the off-shell as well as the on-shell nilpotent symmetry transformations can be succinctly expressed in terms of the conserved ($\dot{Q}_r = \tilde{Q}_r = 0$) charges (Q_r, \tilde{Q}_r) and the generic field $\Psi = x_\mu, p_\mu, c, \bar{c}, b, e$, as

$$s_r \Psi = -i [\Psi, Q_r]_\pm, \quad \tilde{s}_r \Psi = -i [\Psi, \tilde{Q}_r]_\pm, \quad r = b, ab, \quad (2.7)$$

where Q_{ab} and \tilde{Q}_{ab} are the off-shell- and the on-shell nilpotent versions of the anti-BRST charges which can be obtained from their counterpart BRST charges by the substitutions: $c \rightarrow \bar{c}, \bar{c} \rightarrow c$. The explicit expressions for the conserved charges $Q_{(a)b}$ and $\tilde{Q}_{(a)b}$ are not required for our present discussions but they can be found in [24]. The \pm signs, present as the subscripts on the above square brackets, stand for the (anti-)commutators for the

[§]We follow here the notations and conventions adopted by Weinberg [25]. In its totality, the true nilpotent (anti-)BRST transformations $\delta_{(A)B}$ are the product of an (anticommuting) spacetime independent parameter η and the nilpotent transformations $s_{(a)b}$. It is clear that η commutes with all the bosonic (even) fields of the theory and anti-commutes with fermionic (odd) fields (i.e. $\eta c + c\eta = 0, \eta \bar{c} + \bar{c}\eta = 0$).

generic field Ψ being (fermionic)bosonic in nature.

3 Symmetries for the gauge- and (anti-)ghost fields in superfield approach

We begin here with a general three $(1 + 2)$ -dimensional supermanifold parametrized by the superspace coordinates $Z = (\tau, \theta, \bar{\theta})$ where τ is an even (bosonic) coordinate and θ and $\bar{\theta}$ are the two odd (Grassmannian) coordinates (with $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$). On this supermanifold, one can define a 1-form supervector superfield $\tilde{V} = dZ(\tilde{A})$ with $\tilde{A}(\tau, \theta, \bar{\theta}) = (E(\tau, \theta, \bar{\theta}), \Phi(\tau, \theta, \bar{\theta}), \bar{\Phi}(\tau, \theta, \bar{\theta}))$ as the component multiplet superfields. The superfields $E, \Phi, \bar{\Phi}$ can be expanded in terms of the basic fields (e, c, \bar{c}) and auxiliary field (b) along with some extra secondary fields (i.e. $f, \bar{f}, B, \mathcal{B}, \bar{\mathcal{B}}, s, \bar{s}, \bar{b}$), as (see, e.g., [9])

$$\begin{aligned} E(\tau, \theta, \bar{\theta}) &= e(\tau) + \theta \bar{f}(\tau) + \bar{\theta} f(\tau) + i \theta \bar{\theta} B(\tau), \\ \Phi(\tau, \theta, \bar{\theta}) &= c(\tau) + i \theta \bar{b}(\tau) + i \bar{\theta} \mathcal{B}(\tau) + i \theta \bar{\theta} s(\tau), \\ \bar{\Phi}(\tau, \theta, \bar{\theta}) &= \bar{c}(\tau) + i \theta \bar{\mathcal{B}}(\tau) + i \bar{\theta} b(\tau) + i \theta \bar{\theta} \bar{s}(\tau). \end{aligned} \quad (3.1)$$

It is straightforward to note that the local fields $f(\tau), \bar{f}(\tau), c(\tau), \bar{c}(\tau), s(\tau), \bar{s}(\tau)$ on the r.h.s. are fermionic (anti-commuting) in nature and the bosonic (commuting) local fields in (3.1) are: $e(\tau), B(\tau), \mathcal{B}(\tau), \bar{\mathcal{B}}(\tau), b(\tau), \bar{b}(\tau)$. It is unequivocally clear that, in the above expansion, the bosonic- and fermionic degrees of freedom match. This requirement is essential for the validity and sanctity of any arbitrary supersymmetric theory in the superfield formulation. In fact, all the secondary fields will be expressed in terms of basic fields due to the restrictions emerging from the application of the horizontality condition [9,8]

$$\tilde{d} \tilde{V} = d A = 0, \quad d = d\tau \partial_\tau, \quad A = d\tau e(\tau), \quad d^2 = 0, \quad (3.2)$$

where the super exterior derivative \tilde{d} and the super connection 1-form \tilde{V} are defined as

$$\begin{aligned} \tilde{d} &= d\tau \partial_\tau + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}, \\ \tilde{V} &= d\tau E(\tau, \theta, \bar{\theta}) + d\theta \Phi(\tau, \theta, \bar{\theta}) + d\bar{\theta} \bar{\Phi}(\tau, \theta, \bar{\theta}). \end{aligned} \quad (3.3)$$

We expand $\tilde{d} \tilde{V}$, present in the l.h.s. of (3.2), as [¶]

$$\begin{aligned} \tilde{d} \tilde{V} &= (d\tau \wedge d\theta)(\partial_\tau \bar{\Phi} - \partial_\theta E) - (d\theta \wedge d\bar{\theta})(\partial_\theta \bar{\Phi}) + (d\tau \wedge d\bar{\theta})(\partial_\tau \Phi - \partial_{\bar{\theta}} E) \\ &- (d\theta \wedge d\bar{\theta})(\partial_\theta \Phi + \partial_{\bar{\theta}} \bar{\Phi}) - (d\bar{\theta} \wedge d\theta)(\partial_{\bar{\theta}} \Phi). \end{aligned} \quad (3.4)$$

Ultimately, the application of the horizontality condition ^{||} ($\tilde{d} \tilde{V} = dA = 0$) yields

$$\begin{aligned} f(\tau) &= \partial_\tau c(\tau) \equiv \dot{c}(\tau), & \bar{f}(\tau) &= \partial_\tau \bar{c}(\tau) \equiv \dot{\bar{c}}(\tau), & s(x) &= \bar{s}(x) = 0, \\ B(\tau) &= \partial_\tau b(\tau) \equiv \dot{b}(\tau), & b(\tau) + \bar{b}(\tau) &= 0, & \mathcal{B}(\tau) &= \bar{\mathcal{B}}(\tau) = 0. \end{aligned} \quad (3.5)$$

[¶]We have exploited here: $d\tau \wedge d\tau = 0, d\tau \wedge d\theta = -d\theta \wedge d\tau, d\tau \wedge d\bar{\theta} = -d\bar{\theta} \wedge d\tau, d\theta \wedge d\bar{\theta} = +d\bar{\theta} \wedge d\theta$, etc. together with $\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0, \partial_\theta \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_\theta = 0$ and $(d\theta \partial_\theta) \wedge (d\bar{\theta} \partial_{\bar{\theta}}) = -(d\theta \wedge d\bar{\theta})(\partial_\theta \partial_{\bar{\theta}})$, etc.

^{||}It should be noted that the gauge (einbein) field $e(\tau)$ is a scalar potential depending only on a single parameter τ . This is why the curvature $dA = 0$ (because $d\tau \wedge d\tau = 0$). For the 1-form Abelian gauge theory where the gauge field is a vector potential $A_\mu(x)$ (defined through $A = dx^\mu A_\mu$), the 2-form curvature $dA = \frac{1}{2}(dx^\mu \wedge dx^\nu) F_{\mu\nu}$ is not equal to zero and defines the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The insertion of all the above values in the expansion (3.1) leads to the derivation of the (anti-)BRST symmetry transformations for the gauge- and (anti-)ghost fields of the theory. This statement can be expressed, in an explicit form, as given below

$$\begin{aligned} E(\tau, \theta, \bar{\theta}) &= e(\tau) + \theta \dot{c}(\tau) + \bar{\theta} \dot{c}(\tau) + i \theta \bar{\theta} \dot{b}(\tau), \\ \Phi(\tau, \theta, \bar{\theta}) &= c(\tau) - i \theta b(\tau), \\ \bar{\Phi}(\tau, \theta, \bar{\theta}) &= \bar{c}(\tau) + i \bar{\theta} b(\tau). \end{aligned} \quad (3.6)$$

In addition, this exercise provides the physical interpretation for the (anti-)BRST charges $Q_{(a)b}$ as the generators (cf.(2.7)) of translations (i.e. $\text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta)$, $\text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$) along the Grassmannian directions of the supermanifold. Both these observations can be succinctly expressed, in a combined way, by re-writing the super expansion (3.1) as

$$\begin{aligned} E(\tau, \theta, \bar{\theta}) &= e(\tau) + \theta (s_{ab}e(\tau)) + \bar{\theta} (s_b e(\tau)) + \theta \bar{\theta} (s_b s_{ab}e(\tau)), \\ \Phi(\tau, \theta, \bar{\theta}) &= c(\tau) + \theta (s_{ab}c(\tau)) + \bar{\theta} (s_b c(\tau)) + \theta \bar{\theta} (s_b s_{ab}c(\tau)), \\ \bar{\Phi}(\tau, \theta, \bar{\theta}) &= \bar{c}(\tau) + \theta (s_{ab}\bar{c}(\tau)) + \bar{\theta} (s_b \bar{c}(\tau)) + \theta \bar{\theta} (s_b s_{ab}\bar{c}(\tau)). \end{aligned} \quad (3.7)$$

It should be noted that the third and fourth terms of the expansion for Φ and the second and fourth terms of the expansion for $\bar{\Phi}$ are zero because $(s_b c = 0, s_{ab}\bar{c} = 0)$.

Let us now focus on the derivation of the on-shell nilpotent (anti-)BRST symmetry transformations $\tilde{s}_{(a)b}$ of (2.5) in the framework of superfield approach. To this end in mind, we begin with the chiral limit $\theta \rightarrow 0$ of the expansions (3.1) and definition (3.3), as

$$\begin{aligned} (E)|_{(c)}(\tau, \bar{\theta}) &= e(\tau) + \bar{\theta} f(\tau), & (\Phi)|_{(c)}(\tau, \bar{\theta}) &= c(\tau) + i \bar{\theta} \mathcal{B}(\tau), \\ (\bar{\Phi})|_{(c)}(\tau, \bar{\theta}) &= \bar{c}(\tau) + i \bar{\theta} b(\tau), & (\tilde{d})|_{(c)} &= d\tau \partial_\tau + d\bar{\theta} \partial_{\bar{\theta}}, \\ (\tilde{V})|_{(c)}(\tau, \bar{\theta}) &= d\tau (E)|_{(c)}(\tau, \bar{\theta}) + d\bar{\theta} (\Phi)|_{(c)}(\tau, \bar{\theta}). \end{aligned} \quad (3.8)$$

The horizontality condition $(\tilde{d})|_{(c)}(\tilde{V})|_{(c)} = dA = 0$, leads to the following conditions

$$\partial_\tau(\Phi)|_{(c)} = \partial_{\bar{\theta}}(E)|_{(c)} \Rightarrow f(\tau) = \dot{c}(\tau), \quad \partial_{\bar{\theta}}(\Phi)|_{(c)} = 0 \Rightarrow \mathcal{B}(\tau) = 0, \quad (3.9)$$

which emerge from the explicit expression for $(\tilde{d})|_{(c)}(\tilde{V})|_{(c)} = 0$ as given below

$$(\tilde{d})|_{(c)}(\tilde{V})|_{(c)} = (d\tau \wedge d\bar{\theta}) [\partial_\tau(\Phi)|_{(c)} - \partial_{\bar{\theta}}(E)|_{(c)}] - (d\bar{\theta} \wedge d\bar{\theta}) [\partial_{\bar{\theta}}(\Phi)|_{(c)}] \equiv 0. \quad (3.10)$$

It is clear that two of the three extra fields present in the expansion (3.8) are found in (3.9) due to the horizontality restriction. However, the extra field $b(\tau)$ is *not* determined by the above condition. Fortunately, the equation of motion $b(\tau) + \dot{c}(\tau) = 0$, emerging from the Lagrangian density (2.2), comes to our rescue. With these inputs, we obtain the following form of the expansion (3.1) *vis-à-vis* the on-shell nilpotent transformations (2.5):

$$\begin{aligned} (E)|_{(c)}(\tau, \bar{\theta}) &= e(\tau) + \bar{\theta} (\tilde{s}_b e(\tau)), & (\Phi)|_{(c)}(\tau, \bar{\theta}) &= c(\tau) + \bar{\theta} (\tilde{s}_b c(\tau)), \\ (\bar{\Phi})|_{(c)}(\tau, \bar{\theta}) &= \bar{c}(\tau) + \bar{\theta} (\tilde{s}_b \bar{c}(\tau)). \end{aligned} \quad (3.11)$$

The above expansion, together with the help of (2.7), provides the geometrical interpretation for the conserved and on-shell nilpotent BRST charge \tilde{Q}_b as the generator of translation (i.e. $(\partial/\partial\bar{\theta})$) for the *chiral* superfields (3.8) along the $\bar{\theta}$ -direction of the two $(1+1)$ -dimensional chiral super sub-manifold, parametrized by an even variable τ and an odd

variable $\bar{\theta}$. In a similar fashion, one can derive the on-shell nilpotent anti-BRST transformations of (2.5) by exploiting the anti-chiral limit of (3.1). In fact, in the limit $\bar{\theta} \rightarrow 0$, the expansions in (3.1) and definition in (3.3) lead to

$$\begin{aligned} (E)|_{(ac)}(\tau, \theta) &= e(\tau) + \theta \bar{f}(\tau), & (\Phi)|_{(ac)}(\tau, \theta) &= c(\tau) - i \theta b(\tau), \\ (\bar{\Phi})|_{(ac)}(\tau, \theta) &= \bar{c}(\tau) + i \theta \bar{\mathcal{B}}(\tau), & (\tilde{d})|_{(ac)} &= d\tau \partial_\tau + d\theta \partial_\theta, \\ (\tilde{V})|_{(ac)}(\tau, \theta) &= d\tau (E)|_{(ac)}(\tau, \theta) + d\theta (\bar{\Phi})|_{(ac)}(\tau, \theta). \end{aligned} \quad (3.12)$$

It should be noted that, from our earlier consideration, it was found that $b(\tau) + \bar{b}(\tau) = 0$. This relation has been exploited here to replace $\bar{b}(\tau)$ in the expansion of $(\Phi)|_{(ac)}$ by $-b(\tau)$. The horizontality condition $(\tilde{d})|_{(ac)}(\tilde{V})|_{(ac)} = dA = 0$, leads to the following conditions

$$\partial_\tau(\bar{\Phi})|_{(ac)} = \partial_\theta(E)|_{(ac)} \Rightarrow \bar{f}(\tau) = \dot{\bar{c}}(\tau), \quad \partial_\theta(\bar{\Phi})|_{(ac)} = 0 \Rightarrow \bar{\mathcal{B}}(\tau) = 0, \quad (3.13)$$

which emerge from the explicit expression for $(\tilde{d})|_{(ac)}(\tilde{V})|_{(ac)} = 0$ as listed below

$$(\tilde{d})|_{(ac)}(\tilde{V})|_{(ac)} = (d\tau \wedge d\theta) [\partial_\tau \bar{\Phi}]|_{(ac)} - \partial_\theta(E)|_{(ac)}] - (d\theta \wedge d\theta) [\partial_\theta(\bar{\Phi})|_{(ac)}] \equiv 0. \quad (3.14)$$

It can be seen that two of the three extra fields of the expansion (3.12) are found in (3.13). However, the extra field $b(\tau)$ is *not* determined by the condition (3.13). The equation of motion $b(\tau) + \dot{c}(\tau) = 0$, emerging from the Lagrangian (2.2), comes to our help (i.e. $b = -\dot{c}$). With the above insertions, the expansions in (3.12) become

$$\begin{aligned} (E)|_{(ac)}(\tau, \theta) &= e(\tau) + \theta (\tilde{s}_{ab} \dot{c}(\tau)), & (\Phi)|_{(ac)}(\tau, \theta) &= c(\tau) + \theta (\tilde{s}_{ab} \dot{c}(\tau)), \\ (\bar{\Phi})|_{(ac)}(\tau, \theta) &= \bar{c}(\tau) + \theta (\tilde{s}_{ab} \dot{c}(\tau)). \end{aligned} \quad (3.15)$$

The above expansions, together with the help of the general transformations (2.7), provide the geometrical interpretation for the conserved and on-shell nilpotent anti-BRST charge \tilde{Q}_{ab} as the generator of translation (i.e. $(\partial/\partial\theta)$) for the *anti-chiral* superfields (3.12) along the θ -direction of the two $(1+1)$ -dimensional anti-chiral super sub-manifold, parametrized by an even variable τ and an odd variable θ .

4 Symmetries for the target space variables in superfield formalism

In contrast to the horizontality condition that relies heavily on the (super-)exterior derivatives $(\tilde{d})d$ and the (super) one-forms $(\tilde{V})A$ for the derivation of the (anti-)BRST symmetry transformations on the gauge field $e(\tau)$ and the (anti-)ghost fields $(\bar{c})c$, the corresponding nilpotent symmetries for the target fields $(x^\mu(\tau), p_\mu(\tau))$ are obtained due to the invariance of the conserved charge of the theory. To justify this assertion, first of all, we start off with the super expansion of the superfields $(X^\mu, P_\mu)(\tau, \theta, \bar{\theta})$, corresponding to the ordinary target variables $(x^\mu, p_\mu)(\tau)$ (that specify the Minkowski cotangent manifold), as

$$\begin{aligned} X_\mu(\tau, \theta, \bar{\theta}) &= x_\mu(\tau) + i \theta \bar{R}_\mu(\tau) + i \bar{\theta} R_\mu(\tau) + i \theta \bar{\theta} S_\mu(\tau), \\ P_\mu(\tau, \theta, \bar{\theta}) &= p_\mu(\tau) + i \theta \bar{F}_\mu(\tau) + i \bar{\theta} F_\mu(\tau) + i \theta \bar{\theta} T_\mu(\tau). \end{aligned} \quad (4.1)$$

It is evident that, in the limit $(\theta, \bar{\theta}) \rightarrow 0$, we get back the canonically conjugate target space variables $(x^\mu(\tau), p_\mu(\tau))$ of the first-order Lagrangian in (2.1). Furthermore, the number of bosonic fields $(x_\mu, p_\mu, S_\mu, T_\mu)$ do match with the fermionic fields $(F_\mu, \bar{F}_\mu, R_\mu, \bar{R}_\mu)$ so that the above expansion is consistent with the basic tenets of supersymmetry. All the component fields on the r.h.s. of the expansion (4.1) are functions of the monotonically increasing parameter τ of the world-line. As emphasized in Section 2, two most decisive features of the free relativistic particle are (i) $\dot{p}_\mu = 0$, and (ii) $p^2 - m^2 = 0$. In fact, it can be seen that the conserved *gauge* charge $Q_g = \frac{1}{2}(p^2 - m^2)$ couples to the ‘gauge’ (einbein) field $e(\tau)$ in the Lagrangian density L_f to maintain the local gauge invariance ** under the transformations $\delta_g x_\mu = \xi p_\mu$, $\delta_g p_\mu = 0$, $\delta_g e = \dot{\xi}$. For the BRST invariant Lagrangian (2.2), the same kind of coupling exists for the local BRST invariance to be maintained in the theory. The invariance of the condition $(p^2 - m^2 = 0)$ on the supermanifold

$$P_\mu(\tau, \theta, \bar{\theta})P^\mu(\tau, \theta, \bar{\theta}) - m^2 = p_\mu(\tau)p^\mu(\tau) - m^2, \quad (4.2)$$

implies that

$$F_\mu(\tau) = \bar{F}_\mu(\tau) = T_\mu(\tau) = 0, \quad \text{and} \quad P_\mu(\tau, \theta, \bar{\theta}) = p_\mu(\tau). \quad (4.3)$$

In other words, the invariance of the mass-shell condition on the (super)manifolds enforces $P_\mu(\tau, \theta, \bar{\theta})$ to be independent of the Grassmannian variables θ and $\bar{\theta}$. To be consistent with our earlier interpretations for the (anti-)BRST charges in the language of translation generators along the Grassmannian directions $(\theta)\bar{\theta}$ of the supermanifold, it can be seen that the above equation can be re-expressed as

$$P_\mu(\tau, \theta, \bar{\theta}) = p_\mu(\tau) + \theta (s_{ab}p_\mu(\tau)) + \bar{\theta} (s_b p_\mu(\tau)) + \theta \bar{\theta} (s_b s_{ab} p_\mu(\tau)). \quad (4.4)$$

The above equation, *vis-à-vis* (4.3), makes it clear that $s_b p_\mu(\tau) = 0$ and $s_{ab} p_\mu(\tau) = 0$. One of the most important relations, that plays a pivotal role in the derivation of the mass-shell condition $(p^2 - m^2 = 0)$ for the Lagrangian L_f , is $\dot{x}_\mu(\tau) = e(\tau)p_\mu(\tau)$. A simpler way to derive the (anti-)BRST transformations on the target variables is to require the invariance of this central relation on the supermanifold as

$$\dot{X}_\mu(\tau, \theta, \bar{\theta}) = E(\tau, \theta, \bar{\theta})P_\mu(\tau, \theta, \bar{\theta}), \quad (4.5)$$

where $E(\tau, \theta, \bar{\theta})$ is the expansion in (3.6) which has been obtained after the application of the horizontality condition. Insertions of the expansions in (4.1), (3.6) and (4.3) into (4.5) lead to the following explicit equation:

$$\dot{x}_\mu(\tau) + \theta \dot{\bar{R}}_\mu(\tau) + \bar{\theta} \dot{R}_\mu(\tau) + i\theta \bar{\theta} \dot{S}_\mu(\tau) = [e(\tau) + \theta \dot{c}(\tau) + \bar{\theta} \dot{\bar{c}}(\tau) + i\theta \bar{\theta} \dot{b}(\tau)] p_\mu(\tau). \quad (4.6)$$

The equality of the coefficients of the appropriate terms from l.h.s. and r.h.s. yields

$$\dot{x}_\mu = e p_\mu, \quad \dot{\bar{R}}_\mu = \dot{\bar{c}} p_\mu, \quad \dot{R}_\mu = \dot{c} p_\mu, \quad \dot{S}_\mu = \dot{b} p_\mu. \quad (4.7)$$

**Exactly the same kind of coupling exists for the *interacting* 1-form (non-)Abelian gauge theories where the matter conserved current $J_\mu = \bar{\psi} \gamma_\mu \psi$, constructed by the Dirac fields, couples to the gauge field A_μ of the (non-)Abelian gauge theories to maintain the local gauge invariance (see, e.g., [1,2]).

At this crucial stage, we summon one of the most decisive physical insights into the characteristic features of the free relativistic particle which states that there is no action of any kind of force (i.e. $\dot{p}_\mu(\tau) = 0$) on the *free* motion of the particle. Having taken into account this decisive input, we obtain, from (4.7), the following relations

$$\dot{\bar{R}}_\mu \equiv \partial_\tau \bar{R}_\mu = \partial_\tau (\bar{c} p_\mu), \quad \dot{R}_\mu \equiv \partial_\tau R_\mu = \partial_\tau (c p_\mu), \quad \dot{S}_\mu \equiv \partial_\tau S_\mu = \partial_\tau (b p_\mu), \quad (4.8)$$

which lead to

$$\bar{R}_\mu(\tau) = \bar{c} p_\mu, \quad R_\mu(\tau) = c p_\mu, \quad S_\mu(\tau) = b p_\mu. \quad (4.9)$$

The insertions of these values in the expansion (4.1) lead to the derivation of the nilpotent (anti-)BRST transformations ($s_{(a)b}$) on the target space co-ordinate field $x_\mu(\tau)$ as

$$X_\mu(\tau, \theta, \bar{\theta}) = x_\mu(\tau) + \theta (s_{ab} x_\mu(\tau)) + \bar{\theta} (s_b x_\mu(\tau)) + \theta \bar{\theta} (s_b s_{ab} x_\mu(\tau)). \quad (4.10)$$

In our recent papers [20-22] on interacting 1-form (non-)Abelian gauge theories, it has been shown that there is a beautiful consistency and complementarity between the horizontality condition and the requirement of the invariance of conserved matter (super)currents on the (super)manifolds. The former restriction leads to the derivation of nilpotent symmetries for the gauge- and (anti-)ghost fields. The latter restriction yields such transformations for the matter fields. For the case of the free relativistic particle, it can be seen that the invariance of the gauge invariant and conserved charge on the (super)manifolds, leads to the derivation of the transformations on the target field variables. To corroborate this assertion, we see that the conserved and gauge invariant charge $Q_g = \frac{1}{2} (p^2 - m^2)$ is the analogue of the conserved *matter* current of the 1-form interacting (non-)Abelian gauge theory. Since the expansion for $P_\mu(x, \theta, \bar{\theta})$ is trivial (cf. (4.3)), we have to re-express the mass-shell condition (i.e. $e^2(p^2 - m^2) = \dot{x}_\mu \dot{x}^\mu - e^2 m^2$) in the language of the superfields (3.6) and (4.1). Thus, the invariance of the conserved (super)charges on the (super)manifolds is:

$$\dot{X}_\mu(\tau, \theta, \bar{\theta}) \dot{X}^\mu(\tau, \theta, \bar{\theta}) - m^2 E(\tau, \theta, \bar{\theta}) E(\tau, \theta, \bar{\theta}) = \dot{x}_\mu(\tau) \dot{x}^\mu(\tau) - e^2 m^2. \quad (4.11)$$

The equality of the appropriate terms from the l.h.s. and r.h.s. leads to

$$\dot{x}_\mu \dot{\bar{R}}^\mu = m^2 e \dot{\bar{c}}, \quad \dot{x}_\mu \dot{R}^\mu = m^2 e \dot{c}, \quad \dot{x}_\mu \dot{S}^\mu = m^2 e \dot{b}, \quad \dot{R}_\mu \dot{\bar{R}}^\mu = m^2 \dot{c} \dot{\bar{c}}. \quad (4.12)$$

Taking the help of the key relation $\dot{x}_\mu = e p_\mu$, we obtain the expressions for $\dot{R}_\mu, \dot{\bar{R}}_\mu, \dot{S}_\mu$ exactly same as the ones given in (4.7) *for the mass-shell condition $p^2 - m^2 = 0$ to be valid*. Exploiting the no force (i.e. $\dot{p}_\mu = 0$) criterion on the free relativistic particle, we obtain the expressions for $R_\mu, \bar{R}_\mu, S_\mu$ in exactly the same form as given in (4.9). The insertion of these values in (4.1) leads to the same expansion as given in (4.10). This provides the geometrical interpretation for the (anti-)BRST charges as the *translational generators*. It should be noted that the restrictions in (4.5) and (4.11) are intertwined. However, the latter is more physical because it states the invariance of the *mass-shell* condition *explicitly*.

5 Conclusions

We have derived, in our present investigation, the off-shell (as well as on-shell) nilpotent (anti-)BRST symmetry transformations for the target space variables $x^\mu(\tau)$ and $p_\mu(\tau)$ as well as the gauge (einbein) field (e) and the (anti-)ghost fields $(\bar{c})c$ in the framework of augmented superfield approach to BRST formalism. In particular, the target space fields are found to be exactly like the *matter* (e.g. Dirac, complex scalar) fields of the usual 1-form interacting (non-)Abelian gauge theories. This is because of the fact that, as the conserved current, constructed by the matter fields, couples to the 1-form gauge fields of the usual interacting (non-)Abelian gauge theories to maintain the *local* gauge invariance, in a similar fashion, the conserved charge $Q_g = \frac{1}{2}(p^2 - m^2)$ (constructed by the target space variables p_μ or \dot{x}_μ) couples to the gauge (einbein) field $e(\tau)$ (cf. (2.1) or (2.2)) to maintain the local gauge (or equivalently reparametrization) invariance in our present theory. It has been a long-standing problem to derive the nilpotent symmetries for the *matter* fields of an interacting gauge theory in the superfield formulation (except in our very recent endeavours [20-22]). We chose the system of free scalar relativistic particle for our discussion primarily for four reasons. First and foremost, this theory provides one of the simplest *interacting* gauge theory where the reparametrization invariance is also present. Second, this theory is at the heart of the more general reparametrization invariant string theories which are at the fore-front of the modern-day theoretical research in high energy physics. Third, to check the mutual consistency and complementarity between the horizontality condition and the invariance of the conserved charge/current on the (super)manifolds which lead to the derivation of nilpotent symmetries for *all* the fields of the interacting gauge theories (see, e.g., [20-22]). Finally, to corroborate the assertion that the nilpotent symmetries for matter fields (in the present endeavour the target fields) owe their origin to the invariance of the conserved charges/currents on the (super)manifolds. The general approach of the geometrical superfield formalism with the theoretical arsenal of (i) the horizontality condition, and (ii) the invariance of the conserved charges/currents on the (super)manifold can be applied to its multi-pronged physical applications in the context of reparametrization invariant theories such as spinning relativistic particle, various versions of string theories, brane dynamics and higher-spin gauge theories, etc. We intend to pursue the above mentioned future directions to put our general ideas of investigation on a firmer footing. We hope to report on our key results of the above endeavours in our forthcoming publications.

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